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The Theory of Light Diffraction in Thin Anisotropic Medium

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The coupled wave theory of Raman and Nath diffraction is extended to the case of thin anisotropic holographic media with grating vector parallel to the medium boundaries. Solutions for the wave amplitudes, diffraction efficiencies, and angular mismatch sensitivities are given in transmission geometries for the case of dielectric modulation. The main difference of the new results with respect to the expressions valid for isotropic media arises due to the walk-off between the wave-front and energy propagation directions. The difference is particularly important in materials with large birefringence, such as organic crystals, ordered polymers, and liquid crystalline cells.

Keywords: anisotropic materials; thin diffraction gratings

1. INTRODUCTION

Light diffraction in an inhomogeneous and isotropic media has been the subject of investigation for a long time and has been studied by many researchers [1–4]. An exact definition of a thick and thin grating has been given by Gaylord and Moharam [5]. Many approximate methods of solving Maxwell's equations in an inhomogeneous medium to obtain approximate diffraction formulas or relations are known, because it is difficult to obtain the correct solution. Light diffraction in thick media has been studied thoroughly [1,2]. The theoretical efforts to understand light diffraction in thick isotropic media have culminated in the coupled wave theory of Kogelnik [1]. The same theory for anisotropic thick media has been given by Montemezzani and Zgonik [2]. Kojima [3] investigated the problem of diffraction of

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light in phase gratings in absorption less anisotropic materials finding solutions in the Raman-Nath diffraction regime using a phase function method and in the Bragg diffraction regime using the Born approximation in the undepleted pump limit. In [6] presented rigorous coupled wave analysis for isotropic media and for intermediate regime. The theory of light diffraction in acoustically produced thin media (refractive index gratings) has investigated in a series of papers written by Raman and Nath [4], which applies to isotropic materials. Despite the fact that a large fraction of the materials used for thin holography is optically anisotropic, only limited effort has been made to theoretically analyze the diffraction of light in this kind of media. The couple wave theory for thin anisotropic media is not investigated completely.

In this paper we have developed a coupled wave theory valid for anisotropic thin holographic media. The entrance and exit surfaces of the medium are parallel to each other. We treat the case of transmission gratings only, the former being characterized by a diffracted beam exiting the medium through the same surface as the transmitted beam. We solve the coupled wave equations for thin anisotropic transmission gratings, in a method slowly varying amplitudes. The coupled wave equations are solved to give the diffraction efficiency and the angle-mismatch sensitivity. We have considered a medium containing a phase plane holographic grating.

This article is organized in the following way. Section II performs the basic equations. Section 2.1 performs adduction the derivation of the multiple wave equations valid in anisotropic thin media. In Section 2.2 we have derived a solution of the coupled wave equations in an approximation of unslanted, pure refractive index and transmission volume gratings. Theoretical curves and results are shown in Section 2.3. Conclusions are given in Section III.

2. BASIC EQUATIONS

2.1. Derivation of the Multiple Wave Equations

We consider a medium containing a phase (refractive index) grating. Our analysis treats the case of thin anisotropic holograms. An exact definition of a thin grating has been given by Gaylord and Moharam [5] and the conditions to be fulfilled are $Q' = Q/\cos\theta = (2\pi\lambda d)/(n^0\Lambda^2 \cos\theta) < 1$ and $Q'\gamma \leq 1$, where $\gamma = \pi n^1 d/\lambda \cos\theta$ and θ is the angle of incidence inside the grating. In our case of anisotropic materials the refractive index change is n^1 . The other quantities in the two above conditions are the medium thickness d , the wavelength in the

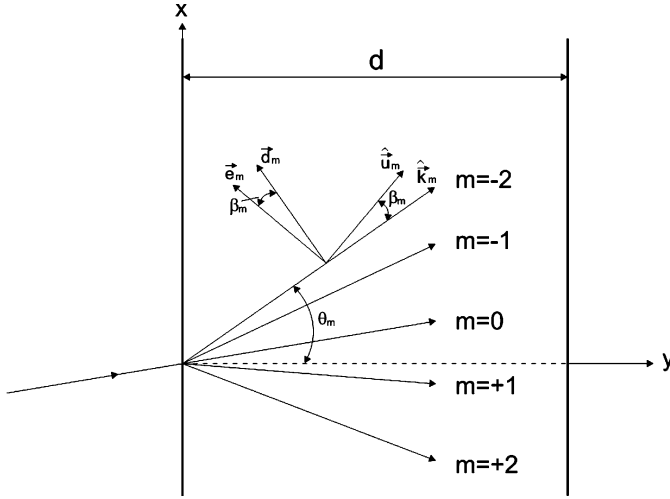


FIGURE 1 Diagram illustrating the wavevectors and angles.

vacuum λ , the average refractive index n^0 , the grating spacing Λ , and the grating wave vector $|\vec{K}| = 2\pi/\Lambda$. We direct the normal to the film surface along the y axis and the x axis is chosen in the plane of incidence and is parallel to the holographic grating vector (Fig. 1). The incident light is assumed to be monochromatic and linearly polarized. We notice that if the two above conditions are not strictly fulfilled the diffraction may be described by a mixture of Bragg and Raman-Nath regimes. In such an intermediate regime the theory presented in this work gives only approximate results and the diffraction would be calculated more precisely by a rigorous coupled wave analysis similar to the one presented earlier for the isotropic case [6].

As shown by Kogelnik [1] for thick gratings it is sufficient to consider the propagation of only two plane waves: signal (incident wave) and pump (diffracted wave). In our case of thin gratings we have one incident wave (signal wave) and multiple diffracted waves. The total electric field amplitude is given by a sum of plane waves, which are propagating along m -orders

$$\vec{E}(\vec{r}, t) = \sum_m \vec{E}_m(\vec{r}) \exp(i\vec{k}_m \vec{r}) \exp(-i\omega t) + c.c., \quad (1)$$

where \vec{E}_m are complex amplitudes of diffracted order m . The wave vectors \vec{k}_m are real and, as usual, are related to the wave-front propagation direction for an eigenpolarization in the crystal. The total

electric field amplitude (1) has to satisfy the time-independent vector wave equation

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) - k_0^2 \vec{\epsilon} \vec{E} = 0 \quad (2)$$

where $\vec{\epsilon}$ is the dielectric tensor that characterize the material refractive index [7], and $k_0 = \omega/c$ is the free-space wave number. From now on the explicit time dependence $\exp(-i\omega t)$ will always be dropped. We consider a transmission grating with refractive index modulation only so the relative permittivity $\vec{\epsilon}$ becomes a real tensor since the constituent material is locally anisotropic. We assume that the relative-permittivity tensor varies sinusoidal and can be expressed as

$$\vec{\epsilon} = \vec{\epsilon}^0 + \vec{\epsilon}^1 \sin(\vec{K}\vec{r}) \quad (3)$$

where the superscripts 0 and 1 denote the constant and the modulated components, respectively. In our unslanted case the grating vector \vec{K} is parallel to grating surface (or perpendicular to the grating planes). We may choose our coordinate system to coincide with the main axes of the optical indicatrix so that the tensors $\vec{\epsilon}^0$ and $\vec{\epsilon}^1$ contains only diagonal elements, that is

$$\vec{\epsilon}^0 = \begin{pmatrix} \epsilon_{xx}^0 & 0 & 0 \\ 0 & \epsilon_{yy}^0 & 0 \\ 0 & 0 & \epsilon_{zz}^0 \end{pmatrix}, \quad \vec{\epsilon}^1 = \begin{pmatrix} \epsilon_{xx}^1 & 0 & 0 \\ 0 & \epsilon_{yy}^1 & 0 \\ 0 & 0 & \epsilon_{zz}^1 \end{pmatrix} \quad (4)$$

The constant and the modulated components of dielectric permittivity have the following components: $\epsilon_{xx}^0 = \epsilon_{\parallel}^0$ and $\epsilon_{xx}^1 = \epsilon_{\parallel}^1$ for the light with polarization parallel to the grating vector, $\epsilon_{yy}^0 = \epsilon_{zz}^0 = \epsilon_{\perp}^0$ and $\epsilon_{yy}^1 = \epsilon_{zz}^1 = \epsilon_{\perp}^1$ for the light with polarization that is perpendicular to the grating vector. We proceed by analyzing the multiple wave equations and we insert Eqs. (3) and (1) into the wave equation (2). We notice that the first term of Eq. (3) can be represented in the following form:

$$\begin{aligned} \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \sum_m \left\{ \exp(i\vec{k}_m\vec{r}) \left[\vec{\nabla} \times (\vec{\nabla} \times \vec{E}_m) - i(\vec{\nabla} \times \vec{E}_m) \right. \right. \\ \left. \left. \times \vec{k}_m - i\vec{\nabla}(\vec{E}_m \times \vec{k}_m) - (\vec{E}_m \times \vec{k}_m) \times \vec{k}_m \right] \right\} \quad (5) \end{aligned}$$

The first term on the right hand side of Eq. (5) contains only second-order derivatives of the wave amplitude and can be neglected applying the slowly varying amplitude approximation. The last term together with the second term of Eq. (2) that contains the contribution of the

nonmodulated dielectric tensors describe the linear propagation of the wave [2]. It is

$$-E_m k_i^2 \sin(\vec{e}_m \vec{k}_m) \vec{d}_m = k_0^2 \vec{\epsilon}^0 \vec{E}_m, \quad (6)$$

where $\vec{E}_m = E_m \vec{e}_m$, $\vec{D}_m = D_m \vec{d}_m$ and \vec{e}_m , \vec{d}_m are unit vectors along the electric-field vector and electric displacement vector, respectively.

The second and third terms on the right-hand side of Eq. (5), which are left, describe the coupling of the waves due to $\vec{\epsilon}^1$. The problem that we are analyzing is interesting for perfect phase matching and for small phase mismatch. In this case we write the momentum conservation equations as

$$\vec{k}_m = \vec{k}_i + m\vec{K} \quad (7)$$

where \vec{k}_i is the wave vector of incident wave. Using the above arguments Eq. (2) transforms in the multiple wave equations

$$\begin{aligned} & \sum_m \left[-i \left\{ (\vec{\nabla} \times \vec{E}_m) \times \vec{k}_m + \vec{\nabla} (\vec{E}_m \times \vec{k}_m) \right\} \right. \\ & \quad \left. - E_m \Delta_m \sin(\vec{e}_m \vec{k}_m) \vec{d}_m \right] \exp(i\vec{k}_m \vec{r}) \\ & = \sum_m k_0^2 \vec{\epsilon}^1 \vec{E}_m \frac{1}{2i} \left[\exp(i\vec{K} \vec{r}) - \exp(-i\vec{K} \vec{r}) \right] \exp(i\vec{k}_m \vec{r}) \end{aligned} \quad (8)$$

where

$$\Delta_m = m^2 K^2 - 2mKk_i \sin(\theta_i) \quad (9)$$

is the phase detuning from the Bragg condition for m diffracted order, θ_i is a internal incident angle. Using some vector algebra the terms on the left-hand side of Eq. (8) can be rewritten as

$$\begin{aligned} (\vec{\nabla} \times \vec{E}_m) \times \vec{k}_m &= (\vec{\nabla} E_m \times \vec{e}_m) \times \vec{k}_m = -\vec{k}_m \times (\vec{\nabla} E_m \times \vec{e}_m) \\ &= -\vec{\nabla} E_m (\vec{k}_m \vec{e}_m) + \vec{e}_m (\vec{k}_m \vec{\nabla} E_m) \end{aligned} \quad (10)$$

and

$$\vec{\nabla} (\vec{E}_m \times \vec{k}_m) = \vec{\nabla} E_m \times (\vec{e}_m \times \vec{k}_m) = \vec{e}_m (\vec{\nabla} E_m \vec{k}_m) - \vec{k}_m (\vec{\nabla} E_m \vec{e}_m) \quad (11)$$

$\vec{\nabla} E_m$ is the gradient of the scalar complex wave amplitude E_m . Summing Eqs. (10) and (11) and multiplying both sides of Eq. (8) with

the unit vector \vec{e}_m one obtains

$$\begin{aligned} & -2i \left| \vec{k}_m \right| \vec{\nabla} E_m \left[\hat{\vec{k}}_m - \vec{e}_m \left(\hat{\vec{k}}_m \vec{e}_m \right) \right] - E_m \Delta_m \sin \left(\vec{e}_m \vec{k}_m \right) \left(\vec{d}_m \vec{e}_m \right) \\ & = \frac{k_0^2}{2i} \left[\vec{e}_m \vec{\varepsilon}^{-1} \vec{e}_{m-1} E_{m-1} - \vec{e}_m \vec{\varepsilon}^{-1} \vec{e}_{m+1} E_{m+1} \right] \end{aligned} \quad (12)$$

The left-hand side vector expression in the square brackets gives a vector that is parallel to the energy propagation direction (Poynting vector) of the m diffracted order [7]. One can write

$$\hat{\vec{k}}_m - \vec{e}_m \left(\hat{\vec{k}}_m \vec{e}_m \right) = g_m \hat{\vec{u}}_m \quad (13)$$

with $\hat{\vec{u}}_m$ being the unit vector along the Poynting vector. Using $\hat{\vec{k}}_m \cdot \hat{\vec{u}}_m = \vec{e}_m \cdot \vec{d}_m = \cos(\beta_m)$ and $\hat{\vec{k}}_m \cdot \vec{d}_m = \vec{e}_m \cdot \hat{\vec{u}}_m = 0$ we get $g_m = \vec{e}_m \cdot \vec{d}_m = \cos(\beta_m)$. We insert Eq. (13) into the multiple wave equations (12) and the final multiple wave equations are given by

$$-2i E'_m k_m g_m \cos(\varphi_m) - E_m \Delta_m g_m^2 = \frac{k_0^2}{2i} [A_{m-1} E_{m-1} - A_{m+1} E_{m+1}] \quad (14)$$

where

$$A_{m-1} = \vec{e}_m \vec{\varepsilon}^{-1} \vec{e}_{m-1} = \vec{e}_{m-1} \vec{\varepsilon}^{-1} \vec{e}_m \quad (15)$$

and

$$A_{m+1} = \vec{e}_m \vec{\varepsilon}^{-1} \vec{e}_{m+1} = \vec{e}_{m+1} \vec{\varepsilon}^{-1} \vec{e}_m \quad (16)$$

are describe the coupling between m , $m-1$ and m , $m+1$ diffracted orders, where the second equalities are valid because the tensor $\vec{\varepsilon}^{-1}$ is symmetric.

$$\varphi_m = \theta_m + \arccos(g_m) \quad (17)$$

is angle between the normal to the surface and Poynting vector for the m -order diffracted beams.

2.2. Solution of the Multiple Wave Equations

Now we can solve the multiple wave equations (14). Let's divide the left- and right-sides of the Eq. (14) on $-2ik_m g_m \cos(\varphi_m)$

$$E'_m - i \frac{g_m \Delta_m}{2k_m \cos \varphi_m} E_m = \frac{k_0}{4n_m g_m \cos \varphi_m} [A_{m-1} E_{m-1} - A_{m+1} E_{m+1}] \quad (18)$$

where $k_m = k_0 n_m$, and n_m is a refractive index of m -order diffracted beam. Let's make the following quantities:

$$\rho_m \equiv \frac{g_m (2Kk_i \sin \theta_i - mK^2)}{2k_m \cos \varphi_m} \quad (19)$$

$$\frac{\xi_{m-1}}{2} = \frac{k_0}{4n_m g_m \cos \varphi_m} A_{m-1} \quad (20)$$

$$\frac{\xi_{m+1}}{2} = \frac{k_0}{4n_m g_m \cos \varphi_m} A_{m+1} \quad (21)$$

and insert Eq. (9) into Eq. (18), we will have

$$E'_m(y) + im\rho_m E_m(y) = \frac{\xi_{m-1}}{2} E_{m-1}(y) - \frac{\xi_{m+1}}{2} E_{m+1}(y) \quad (22)$$

For the solution of the Eq. (22) let's assume in the following form:

$$E_m(y) = E_0 \exp\left(-i\frac{1}{2}m\rho_m y\right) U_m(y) \quad (23)$$

where E_0 is the electric wave amplitude of incident wave at $y = 0$, $U_m(y)$ is the function of y . Substituting the solution (23) in Eq. (22) and apply the following approximation $\rho_{m-1} \approx \rho_{m+1} \approx \rho_m$, we will get

$$\begin{aligned} U'_m(y) + i\frac{1}{2}m\rho_m U_m(y) &= \frac{\xi_{m-1}}{2} U_{m-1}(y) \exp\left(i\frac{1}{2}\rho_m y\right) \\ &\quad - \frac{\xi_{m+1}}{2} U_{m+1}(y) \exp\left(-i\frac{1}{2}\rho_m y\right) \end{aligned} \quad (24)$$

This approximation means that the Bragg mismatch ρ_m is the same for m and its neighbor orders in the equation for m -order diffraction wave. Now let's change the variable in the following way:

$$\chi = \frac{1}{i\rho_m} \left\{ \xi_{m-1} \exp\left(i\frac{1}{2}\rho_m y\right) - \xi_{m+1} \exp\left(-i\frac{1}{2}\rho_m y\right) \right\} \quad (25)$$

By inserting the quantity (25) into Eq. (24) and taking into account that $dU'_m/dy = (dU'_m/d\chi) \cdot (d\chi/dy)$, we can write Eq. (24) in the following form:

$$\begin{aligned} U'_m(\chi) \left\{ \frac{\xi_{m-1}}{2} \exp\left(i\frac{1}{2}\rho_m y\right) + \frac{\xi_{m+1}}{2} \exp\left(-i\frac{1}{2}\rho_m y\right) \right\} &+ i\frac{1}{2}m\rho_m U_m(\chi) \\ &= \frac{\xi_{m-1}}{2} U_{m-1}(\chi) \exp\left(i\frac{1}{2}\rho_m y\right) - \frac{\xi_{m+1}}{2} U_{m+1}(\chi) \exp\left(-i\frac{1}{2}\rho_m y\right) \end{aligned} \quad (26)$$

One can see, that if the function $U_m(\chi)$ is a Bessel function of the order m , hence in Eq. (26) are written recurrence relations between different orders and derivatives of Bessel functions [8]. It means that the solution (23) can be represented in the following form:

$$E_m = E_0 \exp\left(-i\frac{1}{2}m\rho_m y\right) \times J_m\left(\frac{1}{i\rho_m}\left\{\xi_{m-1}\exp\left(i\frac{1}{2}\rho_m y\right) - \xi_{m+1}\exp\left(-i\frac{1}{2}\rho_m y\right)\right\}\right) \quad (27)$$

The electric field amplitude of m -order diffracted beam at $y = d$ is:

$$E_m(y=d) = E_0 \exp\left(-i\frac{1}{2}m\rho_m d\right) \times J_m\left(\frac{1}{i\rho_m}\left\{\xi_{m-1}\exp\left(i\frac{1}{2}\rho_m d\right) - \xi_{m+1}\exp\left(-i\frac{1}{2}\rho_m d\right)\right\}\right) \quad (28)$$

where d is a thickness of medium. Let us define m -order diffraction efficiency as a ratio between m -order output-signal intensity and incident pump intensity:

$$\eta_m = \frac{I_m(y=d)}{I_i(y=0)} = \frac{E_m(y=d)E_m^*(y=d)n_m g_m \cos\theta_m}{E_i(y=0)E_i^*(y=0)n_i g_i \cos\theta_i} \quad (29)$$

where $E_i(y=0)=E_0$, n_i is a refractive index of incident beam, g_i is a projection cosines between \vec{e} and \vec{d} for incident beam. The factor $\cos\theta_m/\cos\theta_i$ is an obliquity term that assures consistent results in a general case when we are interested in the optical energy flow through the input and output surfaces of the medium. The term $n_m g_m/n_i g_i$ has been often overlooked in the literature. Neglecting this term is allowed only in isotropic materials or in anisotropic materials in the case of a configuration fully symmetric.

3. RESULTS

Figure 2 shows the diffraction efficiency as a function of external incident angle for zero-order (transmitted wave), for p and s polarizations. Figure 3 shows the diffraction efficiency as a function of external incident angle for $+1$ -order (diffracted wave), for p and s polarizations. It is seen, that integrated transmission for p polarization wave is less than that for s polarization. Also, the first-order diffraction efficiencies for the s - and p -polarized probe beams are approximately equal. Figure 4 shows the diffraction efficiency as a function of external incident angle for $+2$ -order (diffracted wave), for p and s polarizations.

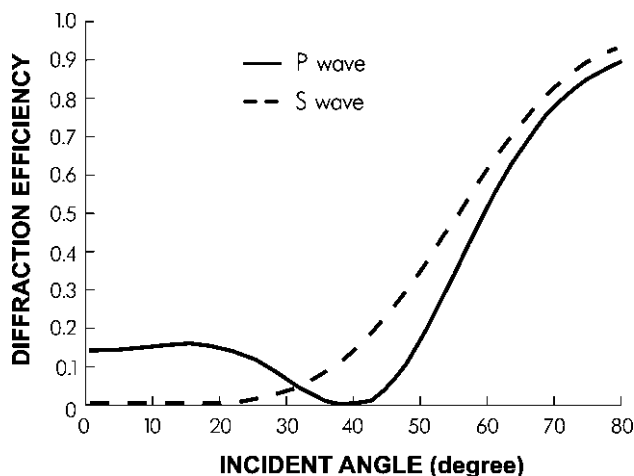


FIGURE 2 Diffraction efficiency vs. external incident angle for zero-order.

Figure 5 shows the diffraction efficiency as a function of external incident angle for +3-order (diffracted wave), for p and s polarizations. From Figure 4 and Figure 5 we can see, that the p -polarization diffraction differs strongly from s -polarization one. The calculations are made for following parameters of a diffraction grating: grating thickness $d = 6.0\mu\text{m}$, grating period $\Lambda = 5.5\mu\text{m}$, incident wavelength $\lambda = 500\text{ nm}$. Average refractive index and modulation are $n^0 = 1.549$

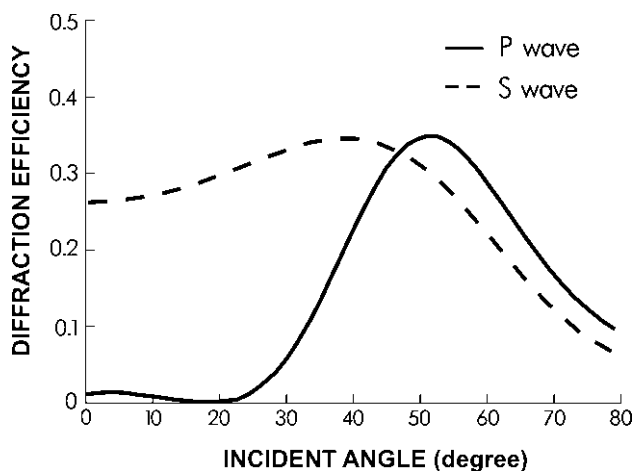


FIGURE 3 Diffraction efficiency vs. external incident angle for +1-order.

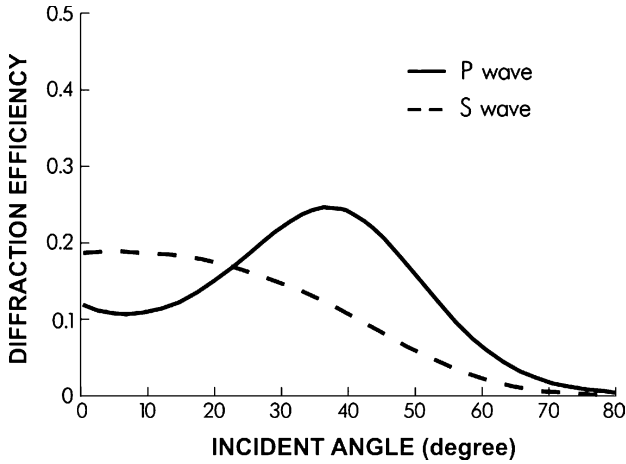


FIGURE 4 Diffraction efficiency vs. external incident angle for +2-order.

and $n^1 = 0.032$ for s polarization, and at $\theta = 0^\circ$ for p polarization refractive index and modulation are $n^0 = 1.637$ and $n^1 = 0.055$, respectively. In this case the conditions, which are defining the diffraction regime, are $Q' = 0.402$ and $Q' \cdot \gamma = 0.612$ for s polarization, $Q' = 0.381$ and $Q' \cdot \gamma = 0.986$ for p polarization. According to [9] we are in Raman-Nath regime. Higher orders of diffraction are also present, but for these parameters of grating they are small, and,

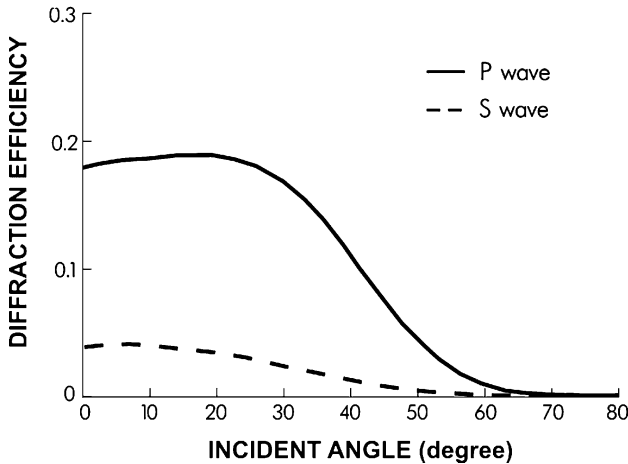


FIGURE 5 Diffraction efficiency vs. external incident angle for +3-order.

consequently, are not shown here. For p wave at normal incidence ($\theta = 0^\circ$) angles of diffraction for $+1$ up to $+5$ orders are following: $\theta_d^{+1} = 3.178^\circ$; $\theta_d^{+2} = 6.337^\circ$; $\theta_d^{+3} = 9.458^\circ$; $\theta_d^{+4} = 12.524^\circ$ and $\theta_d^{+5} = 15.518^\circ$.

All numerical calculations are made with the help of the program "Mathcad 2001 Professional."

4. CONCLUSIONS

So, we have extended the coupled wave theory of Raman and Nath to the case of anisotropic materials, namely for anisotropic thin holographic media. We are applying only the slowly varying amplitude approximation. Solutions for wave amplitudes and diffraction efficiencies have been given for transmission configurations only. From Figures 3, 4, 5 we can see, that p and s wave diffraction efficiency is decrease parallel to increase the diffracted order. And we can see, that after $\theta = 30^\circ$ the diffraction efficiency for p polarization is always higher then s polarization. The s polarized and p polarized waves experience different effective refractive-index modulations, and therefore the coupling between the waves within the grating is different. This difference can lead to a significant change in the diffractive properties of the anisotropic hologram.

The insights provided by the present approach should be particularly important for the analysis of all anisotropic diffraction processes and also for isotropic interaction of light with volume holograms recorded in materials with strong birefringence, such as liquid crystalline cells and ordered polymers.

REFERENCES

- [1] Kogelnik, H. (1969). *Bell Syst. Tech. J.*, **48**, 2909.
- [2] Montemezzani, G. & Zgonik, M. (1997). *Phys. Rev. E*, **55**, 1035.
- [3] Kojima, K. (1982). *Jpn. J. Appl. Phys.*, **21**, 1303.
- [4] (a) Raman, C. V. & Nath, N. S. N. (1935). *Proc. Indian Acad. Sci.*: Pt. I, **2A**, 406; Pt II, **2A**, 413; (1936), Pt. III, **3A**, 75; (1936), Pt. IV, **3A**, 119; (1936), Pt. V, **3A**, 459;
(b) Nath, N. S. N. (1937). *Generalized Theory*, **4A**, 222.
- [5] Gaylord, T. K. & Moharam, M. G. (1981). *Appl. Opt.*, **20**, 3271.
- [6] Moharam, M. G. & Gaylord, T. K. J. (1981). *Opt. Soc. Am.*, **71**, 811.
- [7] Born, M. & Wolf, E. (1980). *Principles of Optics*, 6th ed., Pergamon: Oxford.
- [8] Smirnov, V. I. (1969). *A Course of Highest Mathematics*, Nauka, Moscow.
- [9] Moharam, M. G., Gaylord, T. K., & Magnusson, R. (1980). *Opt. Comm.*, **32**, 19.